Experimental observation of stochastic resonance in a linear electronic array

A. C. H. Rowe
NEC Research Institute, Inc., 4 Independence Way, Princeton, New Jersey 08540-6634

P. Etchegoin*
The Blackett Laboratory, Imperial College of Science, Technology, and Medicine, Prince Consort Road, London SW7 2BZ, United Kingdom

(Received 16 February 2001; revised manuscript received 8 June 2001; published 29 August 2001)

We report the experimental observation of array-enhanced stochastic resonance, spatiotemporal synchronization, and noise-enhanced propagation in a simple coupled linear array of bistable electronic triggers. In addition, we highlight an analogy between charge density wave (CDW) like conductivity and spatiotemporal synchronization in stochastic resonances, several aspects of which are supported by the experimental evidence presented here. This may prove to be important in the understanding of nonlinear conductivity in CDW solids.

DOI: 10.1103/PhysRevE.64.031106 PACS number(s): 05.40.−a, 02.50.−r, 42.66.Si, 87.10.+e

Stochastic resonance (SR) has received a wealth of attention since its introduction by Benzi [1] and co-workers to study periodically recurrent ice ages. Its present applications embrace systems as diverse as laser cavities, electronic circuits, superconducting quantum interference devices, and neural networks [2], to name just a few. The basic underlying idea of SR is *signal amplification via noise*, and it turns out that a myriad of systems in chemistry, physics, biology, and engineering respond to this principle [3]. Despite the activity in this field, the areas of quantum and/or spatiotemporal SR (STSR) are relatively new. It is believed that some of the features of STSR, which emerges in arrays of coupled nonlinear (bistable) oscillators, may be relevant for data transmission in noisy environments, a situation very often encountered in biological systems. As a consequence, STSR may have interesting ramifications in the areas of information theory, computer science, and neural networks which have only just begun to be explored. The majority of the work in STSR consists of theoretical studies of noise enhanced propagation [4], spatiotemporal synchronization [5–7], and coupled oscillators with both time delays [8] and nonlinear couplings [9]. This considerable theoretical input has not been matched by similar progress in experimental studies. Apart from recent work by Löcher and co-workers [10], there are few or no data addressing the problem of spatially distributed nonlinear arrays under the influence of noise. To some extent, the same can be said of spatiotemporal chaos [11,12].

In this article, we demonstrate an experimental realization of a system with STSR in a small cluster. Accordingly, we deal with an experimental realization of STSR in a lattice, as opposed to continuum systems like chemical reactions [13]. Several of the effects theoretically predicted in the recent literature are experimentally confirmed and, in addition, an analogy between coupled bistable oscillators in SR and the physics of charge density waves (CDW's) is introduced. Experimental evidence sustaining this analogy is also presented.

*Email address: p.etchegoin@ic.ac.uk

Nonlinear dynamical systems find a convenient experimental realization in electronic circuits by combining fast (kHz to MHz) dynamics with reliable methods for monitoring the variables. In effect, these are *tailor-made analog computers* to solve specific problems of coupled nonlinear equations in a lattice. The first experimental observation of SR was, in fact, performed with an ac-driven electronic Schmitt trigger [14] (ST hereafter). A bistable ST [15] is a threshold system that has no true SR, in the sense that there is no intrinsic resonance (i.e., its response is frequency independent). It has been argued [16] that noise-induced amplification in a ST should formally be considered as the *dithering effect* (well known in analog-to-digital converters) rather than SR. With this proviso in mind, threshold systems do capture the essential physics of SR and, when coupled in arrays, can be used as a basic unit for wider classes of bistable dynamical systems [16].

The experimental apparatus consists of four linearly coupled ST's, with periodic boundary conditions. The coupling is achieved via analog operational amplifier (op-amp) adders [15]. A schematic view of the ith cell of the chain is shown in Fig. 1(a). The ith ST (ST_i) is composed of an op-amp (triangle) with a voltage divider (R_1 and R_2). The threshold of ST_i is $(V_s R_2)/(R_2+R_1)$, where V_s is the supply voltage [15]. The noninverting input $(-)$ of the op-amp is fed with the sum of an external noise source, an ac signal, and the output voltage of the $(i-1)$th ST. In turn, the output of ST_i feeds $\mathrm{ST}_{(i+1)}$. In this manner, the center of the hysteresis loop of ST_i is influenced by $\mathrm{ST}_{(i-1)}$, with a shift proportional to its total output. Note that the coupling, whose magnitude is determined by an appropriate choice of resistors in the adder, is *unidirectional* in the sense that the output of ST_i directly affects the dynamics of $\mathrm{ST}_{(i+1)}$, but not $\mathrm{ST}_{(i-1)}$. A perturbation introduced at any given site travels only in one direction. This ''directionality'' in the coupling is very useful for studying noise-enhanced propagation while keeping periodic boundary conditions, as we shall show later. Furthermore, the coupling among sites is chosen to be ''negative'' in the sense that it favors an ''antiparallel'' configuration for neighboring sites. If we call the two possible states of a ST *left* (L) and *right* (R), the coupling favors a

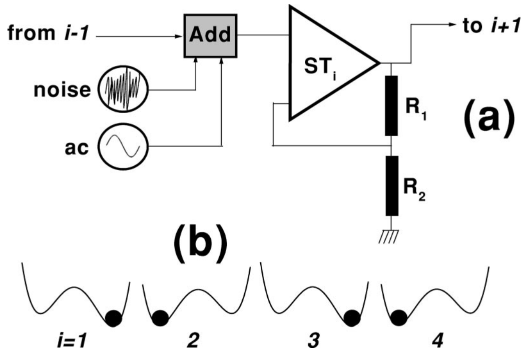

FIG. 1. (a) Schematic circuit for ST_i. The non-inverting input (-) receives a linear combination of (i) the output from $ST_{(i-1)}$, (ii) a Gaussian noise source, and (iii) an ac driving voltage. (b) Mechanical equivalent of the circuit. Each ST_i plays the role of a bistable potential (slightly asymmetric in the ground state due to the presence of the coupling). See text for further details.

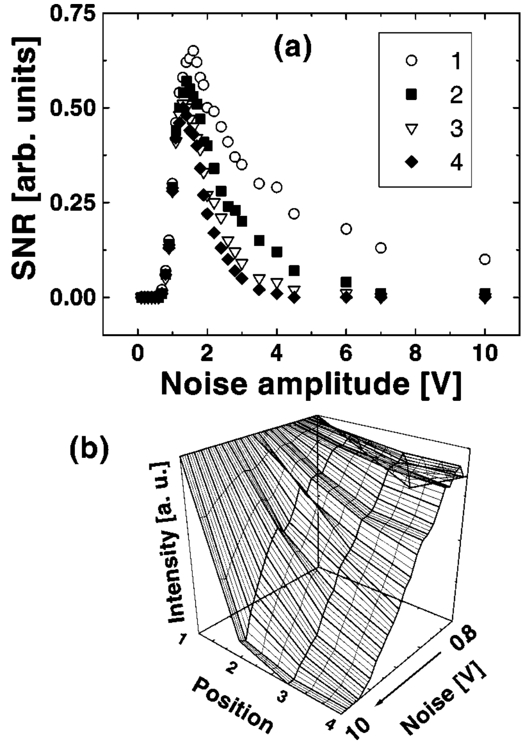

FIG. 2. (a) SNR at ω_{ac} for four different sites in the chain as a function of the noise amplitude. The ac signal is applied to ST_1 only. Note that the further away from ST_1 the faster the SNR decreases as a function of noise. (b) Normalized intensity (to that at ST_1) for the different sites as a function of noise amplitude. Note that the higher the noise, the more localized the perturbation at the first site. For noise amplitudes between ~ 0.8 and 2 V (slightly above the threshold for SR) the signal propagates along the entire array.

configuration with alternating values along the chain ($LRLR\ldots$). In the language of magnetism this would be an antiferromagnetic (AF) chain. We will use the denomination *AF ground state* to refer to this configuration favored by the coupling in the absence of noise. The "mechanical equivalent" is schematically shown in Fig. 1(b), and the tilt of the bistable potential at each site is achieved through the coupling.

The circuit is implemented in practice with Quad-TL084CN op-amps for both the ST's and the adders, and the signals are monitored with both a four-channel 300 MHz bandwidth digital scope and a dual spectrum analyzer. The noise inputs can be either uncorrelated (from site to site) or correlated (global noise), and the spectral characteristics of the noise can be either Gaussian, logarithmic (thermal), or flat (uniform). We shall concentrate here on uncorrelated Gaussian noise only.

Noise-enhanced propagation. In Figs. 2(a) and 2(b), we show the results of the following experiment: one of the ST's only (ST_1) is perturbed with an ac -signal of 300 mV peak to peak (pp) at $\omega_{ac}=1$ kHz. The amplitude is not enough to flip ST_1 and the system remains locked in the ground state. Noise of increasing amplitude is now applied to every site. Above a certain threshold, ST_1 starts responding at ω_{ac} and displays SR. Most importantly, the dynamics of ST_1 are linked to the rest of the lattice via the couplings and the ac response of ST_1 is partly transmitted to its neighbors. There is a range of noise amplitudes within which the perturbation can travel through the whole array. Below this range, the system has no ac response, and is locked into an AF ground state as a result of the couplings. Above it, the noise dominates and long range temporal correlations are rapidly suppressed. This is shown in Fig. 2(a), where the signal-to-noise ratio (SNR) at ω_{ac} is measured for the four sites simultaneously. We observe a typical SR profile [2] at each site with a maximum at ~ 1.8 V. The response is smaller the farther away is the site with respect to ST_1, and at ~ 7 V the signal is essentially confined to ST_1 only. Another way of looking at this effect is to plot the normalized (with respect to ST_1) SNR intensity for each site as a function of noise amplitude as shown in Fig. 2(b). From ~ 0.8 to 2 V the ac signal propagates through the whole array with almost the same SNR as ST_1 but at higher noise amplitudes the SNR can be seen to be concentrated around ST_1 only. This noise-assisted traveling wave is *noise-enhanced propagation* in one dimension, as predicted in Ref. [4].

Array-enhanced SR. We now compare the SNR profiles of the array for different excitation conditions. First, one of the ST's is isolated and its SR profile is measured by driving it with a 1 kHz ac -signal (below threshold) of 300 mV pp, the results of which are shown in Fig. 3(a) as "single" (open triangles). The SNR is then measured at one site in the fully coupled lattice under two different external ac excitations: (i) with the same ac signals at each site [open squares in Fig. 3(a)], and (ii) with ac signals of the same amplitude at each site, but with 180° phase shifts from i to $(i+1)$ (full circles). In the latter case, the external ac signal has the same spatial modulation as the AF ground state of the lattice so the interaction is maximum and this is shown as "coupled" in Fig. 3(a). In the former case, however, the coupling of the external excitation with the ground state is poor, and this is shown as the "out of phase" curve in Fig. 3(a). This experimental result reveals several features, to wit, (i) the coupled and single SNR profiles have differing thresholds [marked with vertical arrows in Fig. 3(a)], as expected from the additional

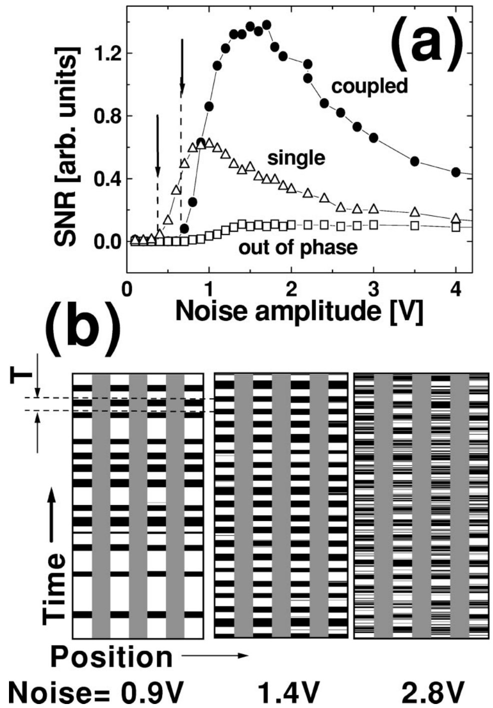

FIG. 3. (a) SNR as a function of noise amplitude for a single isolated ST (open triangles), a linear array with the same ac signal applied to all sites (open squares), and with the ac signal properly coupled to the AF ground state (solid circles). Note (i) the shift in the thresholds between the coupled and single cases, (ii) the enhanced SNR in the coupled case, and (iii) the suppressed SNR in the out-of-phase curve. (b) Three spatiotemporal patterns over a 20 ms period for three different noise amplitudes. Vertical gray areas separate the responses of each of the triggers for clarity. The period of the ac driving potential is shown as T. Note the appearance of spatiotemporal synchronization in the middle plot with the same period as T.

presence of the coupling in the former with respect to the latter; (ii) an enhancement of the SNR profile through linear couplings can be achieved only if the external excitation respects the spatial distribution of the ground state, as seen in the comparison between the coupled and out-of-phase curves; and (iii) the SNR in the coupled system is larger (by roughly a factor of 2) than in the single ST configuration. This is an experimental realization of array-enhanced SR in one dimension as predicted in Ref. [5] where the exact amount of enhancement can be varied by changing the couplings. The experiment allows in principle the exploration of a wide variety of situations in real time.

Spatiotemporal synchronization. Figure 3(b) shows the spatial correlations in the dynamics of the four ST's during 20 ms snapshots at differing noise amplitudes under the same experimental conditions as those labeled "coupled" in Fig. 3(a). We use the convention that ST_1 (ST_2) and ST_3 (ST_4) are displayed as white (light gray) if they are in the L state. By using alternating conventions for i and $(i+1)$, we observe fringes of the same color if the system is AF correlated. Figure 3(b) shows three windows. Each of them is horizontally divided into four vertical strips of the same width, corresponding to each of the ST's, and time evolves vertically.

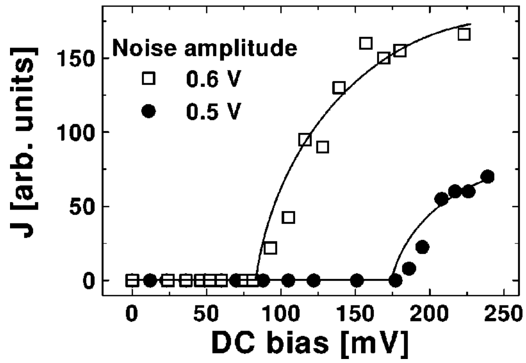

FIG. 4. $J \propto d\phi/dt$ of the coherent flipping of the AF ground state induced by a constant dc potential. Note that the smaller the noise amplitude the larger the threshold for $J \neq 0$. Above the threshold, J increases continuously.

At a noise amplitude of 0.9 V, the ST's display some response in the form of AF correlations, but their period is irregular and only sporadically equal to the external ac driving potential period [1 ms, shown as T in Fig. 3(b)]. At 1.4 V [maximum of the SNR curve in Fig. 3(a)], the system displays clear AF correlations with a period matching T. Finally, at 2.8 V the noise dominates and no AF spatial correlations are observed. The central window in Fig. 3(b) is an experimental example of *spatiotemporal synchronization*. This interplay between nonlinearity, noise, and coupling, is at the heart of the enhanced SNR response of the array in Fig. 3(a) [4,5].

CDW-like conductivity. Consider the following situation: a small amount of noise (below the threshold) is introduced and the array remains in the AF ground state. In addition, a small dc voltage in one direction, say L to R, is applied to each site of the array. The dc potential therefore decreases (increases) the threshold for those sites that were on the L (R) state. The simultaneous action of noise and the DC potential can, accordingly, produce a flip of some sites above a certain threshold. The neighbors can now follow this flip because the coupling favours an overall flipped configuration with respect to the original ground state. The process can be repeated, and an oscillation of average frequency Ω is established—the system goes through a phase of *coherent spatial flipping of the AF ground state* induced by the dc voltage. The larger the dc potential, the more likely the flipping, and the larger the Ω. We can very easily observe this effect in our array; i.e., it is an experimental fact that spatiotemporal synchronization can be induced by a dc field. This effect has not been predicted theoretically, even though the recent theoretical work by Zheng and co-workers [6] shows that it is possible to obtain collective directional transport in a circular array of unidirectionally coupled oscillators. It would be very interesting, in our opinion, to extend their studies to the case of asymmetric potentials. We would like to draw an analogy between the phenomena here and a very similar situation in the physics of CDW's, where the coherent condensate forming the CDW is moved by the action of an external dc potential. In fact, a rigid displacement of the CDW leads to an electric current $j_{CDW} \propto d\phi/dt$, where ϕ is the phase of the CDW order parameter. If $\phi = \Omega t$, then $j_{CDW} \propto \Omega$. This is easily measured in our experiment by monitoring the average Ω induced by a given dc bias in the AF ground state, and it is shown in Fig. 4. In the analogy with CDW-like conductivity, the noise amplitude plays the role of temperature, the threshold for conductivity in Fig. 4 plays the role of the gap in the Pierls instability, and the AF ground state is the CDW condensate (with its amplitude and phase). The threshold in real CDW systems, however, is mainly governed not by the gap, but by impurity pinning. A more accurate analogy with CDW's is also achieved by changing the spectral characteristic of the noise from Gaussian to logarithmic (thermal). This also brings out the interesting question of whether the transition to a ground state in SR can be thought of as a phase transition. It is well known that noise-induced transitions in nonlinear systems can be treated as *phase transitions* in the thermodynamic sense in some specific cases [17]. In fact, noise spectra different from Boltzmann that preserve the general structure of the thermodynamic potentials are known as Tsallis statistics [17], but it is not clear whether thermodynamic concepts can be applied in systems subject to Gaussian or flat noise spectra.

In closing, spatiotemporal synchronization in arrays displaying SR can be observed experimentally in our system and this allows for widespread investigation of the effect of different parameters in real time. In addition, an analogy between the physics of CDW's and SR has been established. We are presently investigating the equivalent effects in SR of current oscillations and Shapiro steps in the dc conductivity of CDW's [18], as well as fault tolerance in noise-enhanced propagation [19].

[1] R. Benzi, A. Sutera, and A. Vulpiani, J. Phys. A **14**, L453 (1981).
[2] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998); A.R. Bulsara and L. Gammaitoni, Phys. Today **49**(3), 39 (1996).
[3] See, for example, *Proceedings of the NATO Advanced Research Workshop: Stochastic Resonance in Physics and Biology*, edited by F. Moss, A. Bulsara, and M. F. Shlesinger (Plenum, New York, 1993).
[4] J.F. Lindner, S. Chandramouli, A.R. Bulsara, M. Löcher, and W.L. Ditto, Phys. Rev. Lett. **81**, 5048 (1998).
[5] J.F. Lindner, B.K. Meadows, W.L. Ditto, M.E. Inchiosa, and A.R. Bulsara, Phys. Rev. Lett. **75**, 3 (1995).
[6] Z. Zheng, G. Hu, and B. Hu, Phys. Rev. Lett. **86**, 2273 (2001).
[7] A.A. Zaikin, J. Kurths, and L. Schimansky-Geier, Phys. Rev. Lett. **85**, 227 (2000).
[8] S. Kim, S.H. Park, and H.B. Pyo, Phys. Rev. Lett. **82**, 1620 (1999).
[9] M.E. Inchiosa and A.R. Bulsara, Phys. Rev. E **52**, 327 (1995).
[10] M. Löcher, G.A. Johnson, and E.R. Hunt, Phys. Rev. Lett. **77**, 4698 (1996); M. Löcher, D. Cigna, and E.R. Hunt, *ibid.* **80**, 5212 (1998).
[11] V. Pérez-Muñuzuri, A.P. Muñuzuri, M. Gómez-Gesteira, V. Pérez-Villar, L. Pivka, and L.O. Chua, Philos. Trans. R. Soc. London, Ser. A **353**, 101 (1995); J.H. Xaio, G. Hu, and Zhilin Qu, Phys. Rev. Lett. **77**, 4162 (1996).
[12] M.N. Lorenzo, I.P. Mariño, V. Pérez-Muñuzuri, M.A. Matías, and V. Pérez-Villar, Phys. Rev. E **54**, R3094 (1996).
[13] O. Kortlüke, V.N. Kuzovkov, and W. von Niessen, Phys. Rev. Lett. **83**, 3089 (1999); B. von Haeften, R. Deza, and H.S. Wio, *ibid.* **84**, 404 (2000).
[14] S. Fauve and F. Heslot, Phys. Lett. **97A**, 5 (1983).
[15] A.S. Sedra and K.C. Smith, *Microelectronic Circuits* (Oxford University Press, Oxford, 1998).
[16] L. Gammaitoni, Phys. Rev. E **52**, 4691 (1995).
[17] C. Tsallis, J. Stat. Phys. **52**, 479 (1988). See also http://tsallis.cat.cbpf.br/biblio.htm
[18] G. Grüner, Rev. Mod. Phys. **60**, 1129 (1988).
[19] R. Perazzo, L. Romanelli, and R. Deza, Phys. Rev. E **61**, R3287 (2000).